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Math 130 - Spring 2025 - Common Final Exam

Print name:____

Section number:______ **Instructor's name:**____________________

Directions:

- This exam has 14 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 103 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator which cannot connect to the internet. The use of any notes or electronic devices other than a calculator is prohibited.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.

Good luck!

Question:	1	2	3	4	5	6	7	8
Points:	6	4	8	3	12	0	4	16
Score:								
Question:	9	10	11	12	13	14		Total
Points:	4	4	8	12	10	12		103
Score:								

1. (6 points) Find the equation of the tangent line to the graph of $y = \dfrac{6}{x^3} + 8x$ at $x = 1$.

2. (4 points) Find the exact value (i.e. without using a calculator) of $p'(2)$ where $p(x) = \dfrac{\ln(x)}{x^3}$.

3. A bookstore is examining the cost (in thousands of dollars) of storing q cubic meters of books and determined it follows the function

$$C(q) = q^4 e^{-2q} + q$$

(a) (4 points) Find the marginal cost function.

(b) (4 points) Find the marginal cost if 3 cubic meters of books are stored. Include units and round your answer to three decimal places.

4. (3 points) The demand function for the latest iPhone is expressed as $q = D(p)$, where q is measured in thousands of units sold, and p is the price in hundreds of dollars. Explain in practical terms the meaning of the statement $D'(6) = -223.8$. Include appropriate units.

5. A local Girl Scout troop has been looking at the sales numbers of their cookies and extrapolated the following table of data, representing the amount $q = D(p)$ of boxes sold as a function of the given price point p (measured in dollars).

p	5.5	5.75	6	6.25	6.5	6.75	7
q	2765	2440	1980	1660	1175	800	430

   (a) (4 points) Find the average rate of change of the demand function $q = D(p)$ on the interval $[5.5, 7]$. Explain the real-life meaning of your answer in an English sentence, including appropriate units.

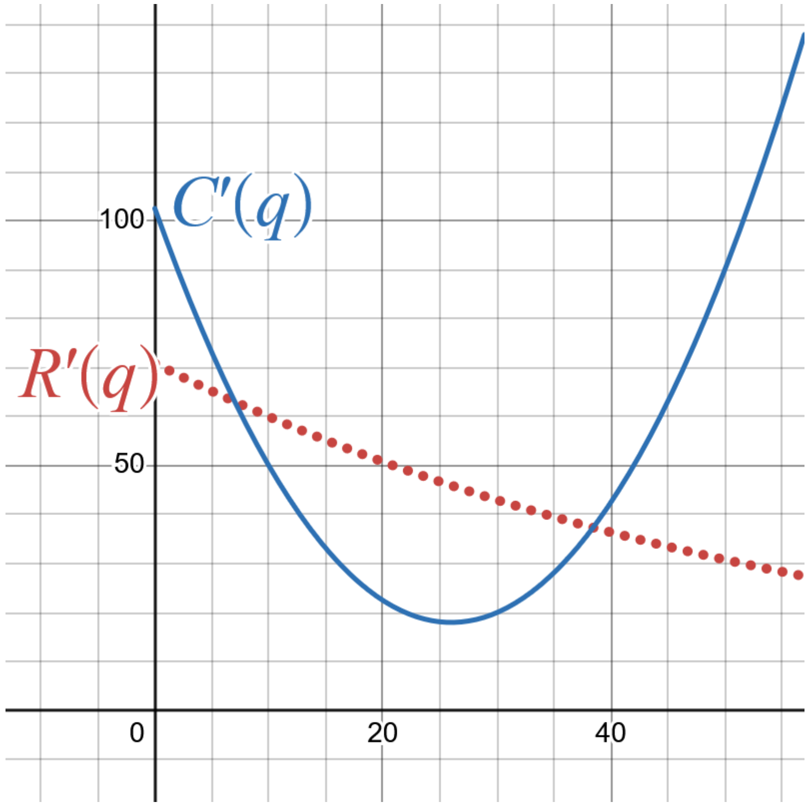
   (b) (6 points) Estimate the elasticity of demand at the price $p = 6$. Explain the real-life meaning of your answer in an English sentence, including appropriate units.

Note: this is still part of question 5, and uses the same demand function $q = D(p)$ given from the table.

(c) (2 points) Is the demand at $p = 6$ elastic or inelastic? If we increased the price, would our revenue increase or decrease? Explain in a sentence.

6. A parent of a newborn child wishes to open a college fund. Suppose that the account is opened today and has a continuous interest rate of 7.1%. Determine the constant amount S that must be invested each year so that the account will contain \$216,000 in 18 years.

7. (4 points) The graph below represents the **MARGINAL** cost $C'(q)$ and **MARGINAL** revenue $R'(q)$ of selling q units of an item.



Use the graph to determine the q-value of maximum profit, explaining your answer in a sentence below.

8. Consider the function

$$f(x) = x^6 - 9x^4.$$

(a) (4 points) Find all critical points of $f(x)$.

(b) (4 points) For each of the critical points found in part (a), determine if it is a local maximum, local minimum or neither. You may use either the first or second derivative test to justify your answer.

Parts (c) and (d) below are a continuation of problem 7, and they refer to the same function

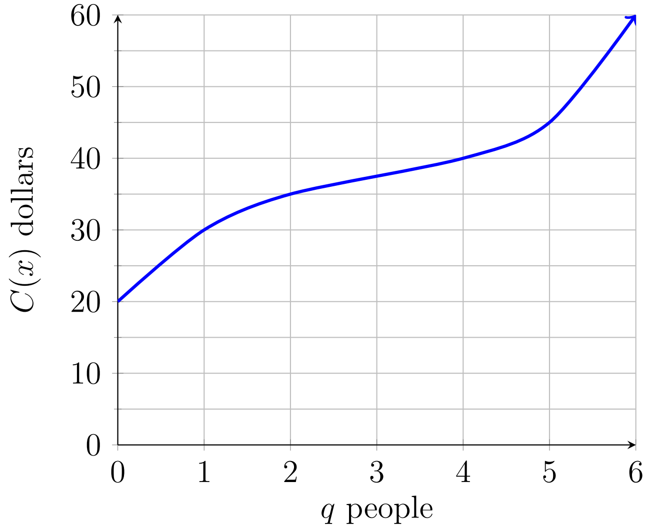
$$f(x) = x^6 - 9x^4$$

used in parts (a) and (b).

(c) (4 points) Find the absolute (or global) maximum and absolute (or global) minimum of $f(x)$ on the interval $-1 \leq x \leq 4$.

(d) (4 points) Find all value(s) of x for which $f''(x) = 0$ and determine whether or not these value(s) are inflection points.

9. (4 points) The graph below represents the total cost $C(q)$ in dollars of providing a service to q people.

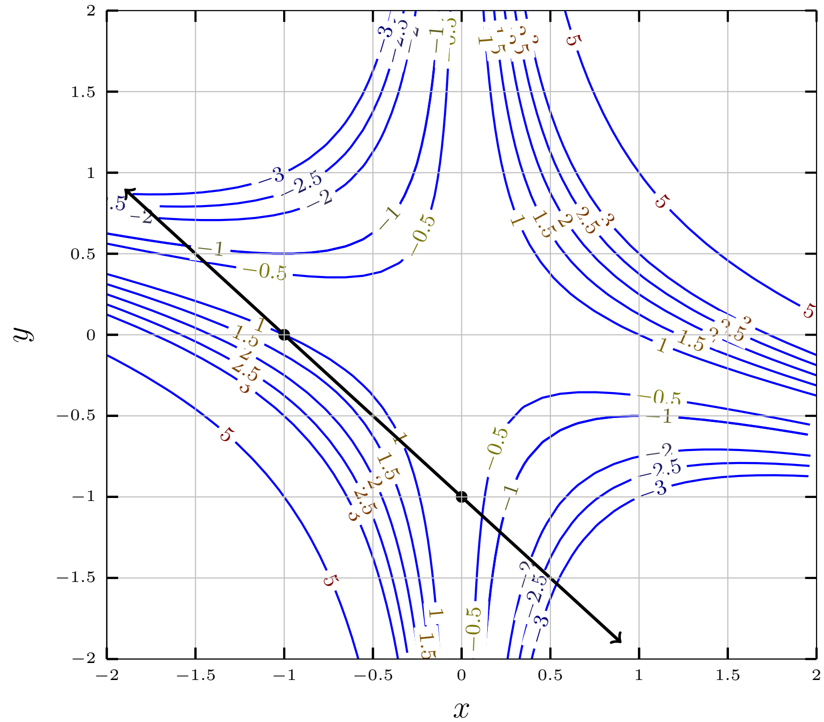


Use the graph to estimate the q-value of minimum **average** cost $a(q)$, explaining your answer in a sentence below.

10. (4 points) Suppose that $f(x)$ is a function such that $\int_3^{11} f(x)\,dx = 24$. If $F(x)$ is an antiderivative of $f(x)$ and $F(3) = -15$, find the value of $F(11)$.

11. A contour plot of $f(x, y) = x^2 + 4xy$ and the graph of the line $x + y = -1$ are given below. We wish to

$$\begin{aligned} \text{optimize} \quad & f(x,y) = x^2 + 4xy \\ \text{subject to} \quad & x + y = -1 \end{aligned}$$



(a) (4 points) Use the graph to estimate the maximum and minimum values of $f(x, y)$ subject to the constraint. From the bubble below, choose all that apply, filling the blanks in as necessary. You do not have to do any calculation.

◯ There is no maximum value subject to the constraint.

◯ Maximum of approximately ____________ at $(x, y) \approx$ ____________

◯ There is no minimum value subject to the constraint.

◯ Minimum of approximately ____________ at $(x, y) \approx$ ____________

(b) (4 points) Write out the system of equations that you would solve to find the (x, y) points giving maximimum or minimum values of f subject to this constraint. You do not have to solve the system.

12. Let $h(x,t) = 3e^{xt}$, and compute the following partial derivatives.

    (a) (4 points) $h_t(x,t)$

    (b) (4 points) $h_{tt}(x,t)$

    (c) (4 points) $h_{tx}(x,t)$

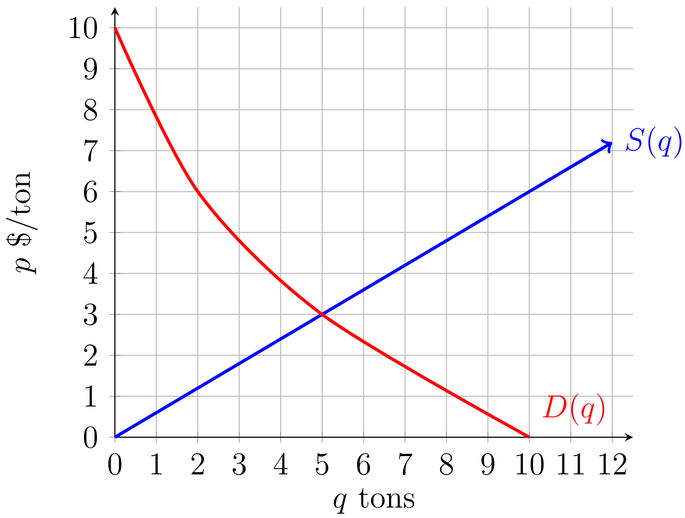
13. Let $f(x, y) = x^3 + y^3 - 6x^2 - 12y + 10$.

    (a) (4 points) Find all first- and second-order partial derivatives of f.

    (b) (3 points) Use your partial derivatives to find the critical points of f. Show your work.

    (c) (3 points) Use your partial derivatives to identify each critical point you found as a local minimum, a local maximum, or neither using the second derivative test. Show your work.

14. The graph below shows supply and demand graphs for a good.



(a) (4 points) Use a Riemann sum with $n = 5$ subintervals to estimate $\int_0^5 D(q)\, dq$. Specify whether you're using left or right sums.

(b) (2 points) Use geometry to find an exact value for $\int_0^5 S(q)\, dq$.

(c) (2 points) What is the equilibrium price and quantity for this market? $(p^*, q^*) =$______________

(d) (1 point) Shade the region on the graph which corresponds to the consumers surplus.

(e) (3 points) Use the at least one of the values you've computed in parts (a) and (b) to estimate the value of the consumer surplus.

Formula Page

Derivatives:

- $\frac{d}{dx}\left(cf(x)\right) = cf'(x)$
- $\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$
- $\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f'(x)g(x) + f(x)g'(x)$
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
- $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$
- $\frac{d}{dx}(c) = 0$, if c is a constant
- $\frac{d}{dx}(mx+b) = m$, where m, b are constants
- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$
- $\frac{d}{dx}\left(a^x\right) = a^x \cdot \ln(a)$
- $\frac{d}{dx}\left(e^x\right) = e^x$
- $\frac{d}{dx}\left(e^{kx}\right) = k \cdot e^{kx}$, if k is a constant
- $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

Other things:

- Quadratic formula:
  $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Future and present value, yearly compounding: $FV = PV \cdot (1+r)^t$
- Future and present value, continously compounding: $FV = PV \cdot e^{rt}$
- Elasticity: $E = \left|\frac{p}{q} \cdot \frac{dq}{dp}\right|$
- Consumer surplus: $\int_0^{q^*} D(q)\, dq - p^* q^*$
- Producer surplus: $p^* q^* - \int_0^{q^*} S(q)\, dq$
- Present value of income stream $S(t)$:
  $PV = \int_0^M S(t) e^{-rt}\, dt$
- Future value of income stream $S(t)$:
  $FV = e^{rM} \cdot PV = \int_0^M S(t) e^{r(M-t)}\, dt$
- Discriminant:
  $D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - (f_{xy}(a,b))^2$